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Дигитални репозиторијум Рударско-геолошког факултета Универзитета у Београду

[ДР РГФ]

Soil oscillation law parameter determination with the application of Lagrange's theorem at "Kovilovača" open pit | Suzana Lutovac, Nebojša Vidanović, Čedomir Beljić, Zoran Gligorić | Mining and Metallurgy Engineering Bor | 2015 | |

10.5937/MMEB1503053L

<http://dr.rgf.bg.ac.rs/s/repo/item/0003103>

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## SOIL OSCILLATION LAW PARAMETER DETERMINATION WITH THE APPLICATION OF LAGRANGE'S THEOREM AT THE "KOVILOVAČA" OPEN PIT\*\*

### Abstract

Large scale mining as a method of extraction of mineral raw materials have a growing application with the aim to increase the quantity of blasted mass, and to reduce costs of production as well. Increasing the quantity of blasted mass requires the use of large amounts of explosives, leading to an increase in the negative effects of mining. Under negative effects of mining the seismic effect of blasting, effect of air waves, sound effect, scattering the blasted rock mass, and similar are assumed.

In order both to evaluate and control the seismic effect of blasting, as well as to plan it, the determination of soil oscillation law is required, with the strike: mine field - facilities to be protected. One of the most commonly used equations is that of M.A. Sadovski defining the law of alteration in the oscillation velocity of the soil depending on distance, explosive amount, and conditions of blasting and geologic characteristics of the soil, being determined on the basis of test blasting for the specific work environment. The Sadovsky equation is determined based on trial minings for a specific work environment.

In this paper, a special attention was paid to the seismic effect of blasting. In accordance with this, an analysis of the method for determining the soil oscillation law parameters, proposed by the Russian professor M.A. Sadovsky, was performed. For determination of parameters in the Sadovsky equation, in addition to the usual model - the least squares method, one more model is shown with the use of Lagrange's theorem. Thereby, it has been stated that both models can be used to calculate the oscillation velocity of the rock mass.

**Keywords:** working environment, blasting, seismic effect, oscillation velocity, soil oscillation law

### 1 INTRODUCTION

As the relation between the soil oscillation velocity and basic parameters affecting its magnitude, being: the amount of explosive, a distance from the blast site, characteristics of the rock material and a type of blasting, the equation of M.A. Sadovski, where the oscillation velocity  $v$  is given in the form of the function, is most frequently used:

$$v = K \cdot R^{-n},$$

where  $R$  is a reduced distance, and  $K$  and  $n$  parameters conditioned by soil charac-

teristics and blasting conditions, thereby  $v$  is the decreasing convex function of the variable  $R$ .

By the application of the law of rock mass oscillation while blasting, the determination of the soil oscillation velocity is enabled for each blast operation in advance, thus blasting is, as regards seismic effect, under control, which gives an opportunity to plan the magnitude of shock waves for each future blast operation [2].

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\*\* This paper has resulted from the Project No. 33025 and 33029 financed by the Ministry of Education of the Republic of Serbia realized by the Faculty of Mining and Geology in Belgrade

In this way the adverse blasting effects are reduced. The adverse effects of blasting imply, in addition to the seismic ones, those of air blast waves, fly rock, etc. Thus production efficiency is increased and, at the same time, construction and mining facilities, as well as the environment in the vicinity of the blast site are protected.

## 2 THE SOIL OSCILLATION LAW

To establish the correlation between the oscillation velocity and three basic parameters affecting its size: the explosive quantity, properties of rock material and the distance, there have been developed several mathematical models in the world. One of most frequently used models, i.e. equations, is the equation of Sadovski defining the law on velocity alteration of soil oscillation depending on the distance, the explosive quantity, and the way of blasting [10]. The law defined in this way offers the possibility to determine the seismic effect of blasting towards a facility or a settlement, whereby the connection, between the velocity of soil oscillation and consequences that can affect facilities, is used.

The equation of M.A Sadovski is given in the form:

$$v = K \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} \quad (1)$$

where:

v - velocity of soil oscillation [cm/s]

K - coefficient conditioned by soil characteristics and blasting conditions determined by terrain surveying,

$$v = K_v \cdot \left( \frac{\sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}}}{r} \right)^{-n} = K_v \cdot \left( \sqrt[3]{\frac{3}{4 \cdot \pi}} \right)^{-n} \cdot \left( \frac{\sqrt[3]{Q}}{r} \right)^{-n} = K_v \cdot K_1 \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot R^{-n}$$

Whereas:

$$\left( \sqrt[3]{\frac{3}{4 \cdot \pi}} \right)^{-n} = K_1; K_v \cdot K_1 = K; \frac{r}{\sqrt[3]{Q}} = R$$

whereby:

n - exponent, conditioned by soil properties and mining conditions and determined by field measurements as well,

r - distance from the blast site to the monitoring point [m],

Q - amount of explosive [kg].

### 2.1 Derivation of equation of rock mass oscillation law

The equation of Sadovski has been derived from the condition: if the radius of charge and the distance from the blast site to the monitoring point increase in the same or approximately the same ratio, the soil oscillation velocity remains the same [1], i. e. that is :

$$v = K_v \cdot \left( \frac{r_o}{r} \right)^n, \quad (2)$$

The radius of the explosive charge  $r_o$  and the amount of explosive Q are related by the equation:

$$Q = \frac{4}{3} \cdot \pi \cdot r_o^3,$$

from where there is:

$$r_o = \sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}}. \quad (3)$$

By replacing the value  $r_o$  from the equation (3), in the equation, the following is obtained (2):

R - reduced distance is the distance from blasting point to observation point reduced to a quantity of explosive and given in the following form  $R = \frac{r}{r_o}$

Thus, it was obtained the oscillation law of rock mass, i.e. the equation of Sadovski in the form:

$$v = K \cdot R^{-n} \quad (4)$$

## 2.2 Models of determination the soil oscillation law parameters

There are two parameters  $K$  and  $n$  in the equation (4) which should be determined for the specific work environment and by particular blasting conditions. With regard to the characteristics of the rock mass oscillation law, it is possible to determine the parameters  $K$  and  $n$  in a number of ways, i.e. models, thereby using the values obtained by experimental measurements.

### ◆ Determination of parameters by model 1

The smallest square method is mainly used to obtain the parameters  $K$  and  $n$  which represents a common model [3].

### ◆ Determination of parameters according to model 2

In the rock mass oscillation velocity law given by equation (4) in the form:

$$v = K \cdot R^{-n} \quad (4)$$

parameter  $n$  can be determined by successive approximations applying the Lagrange's theorem [8].

If rock mass oscillation velocity law given in equation (4) we differentiate by  $R$ , the following is obtained obtain:

$$v' = -n \cdot K \cdot R^{-n-1},$$

i.e.

$$v' = -\frac{n \cdot v}{R} \cdot K \cdot R^{-n}. \quad (5)$$

Bearing in mind (4), equation (5) is reduced to:

$$v' = -\frac{n \cdot v}{R} \quad (6)$$

From (6) the following is found:

$$n = -v' \cdot \frac{R}{v} \quad (7)$$

If the derivative  $v'$  at some point  $R_c$  could not be found then from the equation (7),  $n$  can be determined. Value of derivative  $v'$  can be determined using one of the formulas for numerical differentiation. For this purpose the Stirling's formula is frequently used in case of equidistant values for variable  $R$ . Here it is necessary that  $h$  is a small number, which is not the case with blasting operations in mining engineering.

Previous studies have shown that the value of the parameter  $n$  moves principally in the interval from 1 to 3, most often at an interval from 1 and 2.

To determine the parameter  $n$ , the Lagrange's theorem will be used, which states:

Let  $f(x)$  be a function continuous to  $[a, b]$  and differentiable on an interval  $(a, b)$ . Then there is at least one  $c$ ,

$$a < c < b,$$

for which it is:

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

In this case, given that the velocity is continuous, decreasing and convex function on  $[R_1, R_s]$ , and differentiable on an interval  $(R_1, R_s)$ , so the Lagrange's theorem is valid for it:

$$\frac{v(R_s) - v(R_1)}{R_s - R_1} = v'(R_c),$$

$$R_1 < R_c < R_s. \quad (8)$$

Bearing in mind the formulas (4) and (5), formula (8) is reduced to:

$$\frac{k \cdot R_s^{-n} - k \cdot R_1^{-n}}{R_s - R_1} = -k \cdot n \cdot R_c^{-n-1}, \quad (9)$$

namely to:

$$\frac{R_s^{-n} - R_1^{-n}}{R_s - R_1} = -n \cdot R_c^{-n-1},$$

$$R_1 < R_c < R_s. \quad (10)$$

From the equation (10) the following is obtained:

$$R_c = \left( \frac{n \cdot (R_s - R_1)}{R_1^{-n} - R_s^{-n}} \right)^{\frac{1}{n+1}}, \quad (11)$$

$R_1 < R_c < R_s.$

In practice it may be taken that  $R_1$  is the smallest reduced distance, while  $R_s$  is the greatest reduced distance observed during blasting operations, i.e. measurements.

In the formula (8), the values  $v(R_s)$ ,  $v(R_1)$ ,  $R_s$  and  $R_1$  we take from the table of experimental data, so that we have:

$$\frac{v_s - v_1}{R_s - R_1} = v' \cdot R_c^{-n} \quad (12)$$

In this manner we find a derivative  $v'(R_c)$  at point  $R_c$ , whereas  $R_c$  is given using formula (11).

In practice the formula (8) is used so that the values  $v(R_s)$  and  $v(R_1)$  are taken from tables of the experimental data obtained for corresponding reduced distances  $R_s$  i  $R_1$ , whereas for  $R_s$  the greatest reduced distance is taken, while for  $R_1$  the smallest reduced distance is taken for each observed table.

To find the value  $v'(R_c)$ , in accordance with the formula (8), the formula will be used:

$$v' \cdot R_c^{-n} = \frac{v_s - v_1}{R_s - R_1}, \quad (13)$$

whereas:

$v_s$  – registered rock mass oscillation velocity for the greatest reduced distance  $R_s$ ,

$v_1$  – registered rock mass oscillation velocity for the smallest reduced distance  $R_1$ .

In this way  $v'(R_c)$  is obtained, and then  $R_c$  is determined according to formula (11). In the formula (11) in determining the value  $R_c$  the parameter  $n$  appears. For an initial approximate value of the parameter  $n$ , it will be assumed that:

$$n = n_0 = 1,5.$$

Now, according to the formula (11), for  $n = 1,5$ , the following is:

$$R_c = \left( \frac{1,5 \cdot (R_s - R_1)}{R_1^{-1,5} - R_s^{-1,5}} \right)^{\frac{1}{2,5}}, \quad (14)$$

where the obtained value  $R_c$  lies between values  $R_j$  i  $R_{j+1}$  given in the table of experimental data, which means:

$$R_j < R_c < R_{j+1}.$$

The value  $v_c = v(R_c)$  should be obtained with the value for  $R_c$  determined in such a manner, whereas:

$$v_j < v_c < v_{j+1}.$$

To find the value  $v_c = v(R_c)$ , the interpolation formula will be applied:

$$v_c = v_j + \frac{v_{j+1} - v_j}{R_{j+1} - R_j} \cdot (R_c - R_j) \quad (15)$$

With the value  $v_c$  found in such a way, in accordance with the formula (7), the parameter  $n$  is obtained as:

$$n = -v' \cdot \frac{R_c}{v_c}, \quad (16)$$

which will be marked with  $n_1$ , so according to the formula (8):

$$n_1 = -v' \cdot \frac{R_c}{v_c}. \quad (17)$$

Now, with value  $n_1$  found like this, the formula (11) is used to determine the new values of  $R_c$ , where  $n_1$  is put instead of  $n$ , where the previous procedure is continued for finding a new approximate value  $n_2$  of parameter  $n$ .

In practice, after several iterations, a satisfactory value of the parameter  $n$  is obtained.

The value of parameter  $K$  we obtain from the formula (4), whereas:

$$K = v \cdot R^n \quad (18)$$

In the equation (18) for  $n$ , the value obtained in the previous way is taken.

In so doing, data for pairs  $(R_m, v_m)$ ,  $m = 1, 2, \dots, N$ , are used from the table of experimental data, so that:

$$K_1 = v_1 \cdot R_1^n$$

$$K_2 = v_2 \cdot R_2^n$$

...

$$K_N = v_N \cdot R_N^n,$$

whereby, for parameter  $K$ , the arithmetic mean is used, namely:

$$K = \frac{K_1 + K_2 + \dots + K_N}{N}. \quad (19)$$

In this way, the model for solving the rock mass oscillation law is determined, taking:

$$v = K \cdot R^{-n}$$

which is valid for given environment, where blasting operations were performed.

### 3 DEFINING THE STATISTICAL CRITERIA

To evaluate the degree of correlation between recorded (measured) and calculated data in this paper, the *coefficient of linear correlation*  $r$  [5] was used between the logarithm of reduced distance  $R$  and the logarithm of the oscillation velocity  $v$ . Additionally, it was also taken into account the *curvilinear dependency index*  $\rho$  [4] between the reduced distance  $R$  and the oscillation velocity  $v$ .

The evaluation of the relationship degree of two variables [8] to values of the curvilinear dependency index  $\rho$  is given in the following survey:

$0.0 < \rho < 0.2$  - none or highly poor correlation,

$0.2 < \rho < 0.4$  - poor correlation,

$0.4 < \rho < 0.7$  - significant correlation,

$0.7 < \rho < 1.0$  - strong or highly strong correlation.

The same is valid for the absolute value of linear correlation coefficient  $r$ .

As a convenience measure of the obtained functional relationship for the given

experimental data, the criterion „ $3S$ “ was also used [6]. This criterion uses the squares of differences between the obtained experimental data and the calculated ones for oscillation velocities of  $v$ . If those differences are one after another  $\varepsilon_1, \varepsilon_2 \dots \varepsilon_N$ , then it is :

$$S = \sqrt{\frac{\varepsilon_1^2 + \varepsilon_2^2 + \dots + \varepsilon_N^2}{N}} \quad (20)$$

According to this criterion, for the evaluation of convenience of the obtained functional correlation, the following relations are valid:

- if it is  $|\varepsilon_{\max}| \geq 3S$ , the obtained functional correlation is rejected as unfavorable,
- if it is  $|\varepsilon_{\max}| < 3S$ , the functional correlation is accepted as a good one.

## 4 REVIEW OF MASSIVE BLASTING AT THE “KOVILOVAČA“ OPEN PIT - DESPOTOVAC

### 4.1 General characteristics of the “Kovilovača” open pit

The “Kovilovača“ limestone deposits have an exceptionally simple geological structure. Limestones of this area are massive or layered with layers of thickness from 0.20 to 0.80 m, the direction of propagation is NE-SW and a slope of about 42° towards the southwest. These rocks, in engineering and geological terms, belong to a group of associated rocks, which are cracked and carstified [7]. During previous exploitation and exploitation works, no significant burst deformations, which would significantly influence the process of exploration and exploitation, were noted in the deposit. Only after blasting, blocks of rocks 0.50 m<sup>3</sup> occur, which could be attributed to the effect of small faults and carstified cracks in the deposit.

By examination the physical and mechanical properties of the working environment, the following values are obtained:

- angle of internal friction	$\phi = 31^{\circ} 35'$
- cohesion	$C = 138.33 \text{ [kN/m}^2\text{]}$
- volume weight	$\gamma = 26.26 \text{ [kN/m}^3\text{]}$
- compressive strength	$\sigma_p = 808.08 \text{ [daN/cm}^2\text{]}$
- tensile strength	$\sigma_z = 75.90 \text{ [daN/cm}^2\text{]}$
- velocity of longitudinal elastic waves	$c_p = 6,661.00 \text{ [m/s]}$
- velocity of transverse elastic waves	$c_s = 2,852.67 \text{ [m/s]}$
- dynamic elasticity modulus	$E_{din} = 62.46 \text{ [GN/m}^2\text{]}$
- dynamic Poisson's ratio	$\mu_{din} = 0.39 \text{ [GN/m}^2\text{]}$

#### 4.2 Blasting Method

Measurements of seismic shocks at the "Kovilovača" open pit were performed during blasting, conducted for the purpose of deposit exploitation [9]. Two blasting operations were performed.

Balkanit 60/1500, detonex 65/1500 and ANFO 70/1500 were used as explosives. Activation of explosives in the borehole was performed using the nonel detonators with retardations of 500 ms in the borehole, while

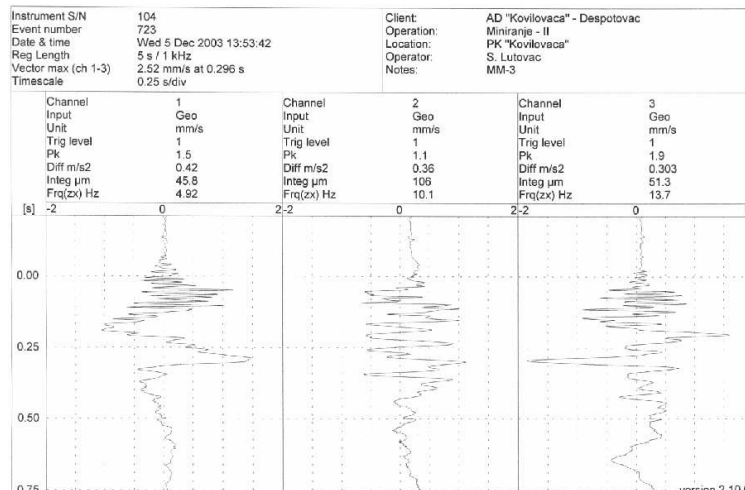
the retardation between boreholes on the surface was 25 ms and 42 ms. Activating of nonel tube was performed using electric detonator.

Basic data related to the number of boreholes ( $N_b$ ), the overall explosive amount ( $Q_{uk}$ ), the maximal explosive amount by deceleration interval ( $Q_i$ ), overall borehole depth ( $L_{uk}$ ), and average stemming length ( $L_{pc}$ ), are presented in Table 1.

**Table 1** Survey of blasting parameters

Blasting	$N_b$	$Q_{uk}$ [kg]	$Q_i$ [kg]	$L_{uk}$ [m]	$L_{pc}$ [m]
I	34	3623.0	108.0	842.0	2.2
II	7	3264.0	92.0	918.0	2.2

The record of soil oscillation velocity is shown in Fig.1 for blasting number II - measuring point 3



**Figure 1** Image of soil oscillation velocity for blasting II-MM3

### 4.3 Calculation of soil oscillation law parameters

Values of distances from blast sites to monitoring points  $r$ , the amount of explosive  $Q$ , calculated values of reduced distances  $R$ , recorded values of soil oscillation

velocities by components  $v_t, v_v, v_l$  and resulting oscillation velocities  $v_{rez}$  for blasting from I to II of totally ten measuring points MM are given in Table 2.

**Table 2** Survey of blasting parameters and measurement results

No	Blasting	MM	$r$ [m]	$Q$ [kg]	$R$	$v_t$ [cm/s]	$v_v$ [cm/s]	$v_l$ [cm/s]	$v_{rez}$ [cm/s]
1	I	MM-1	335.0	3,623.0	21.8117	0.1460	0.0517	1.2000	1.2100
2	I	MM-3	850.0	3,623.0	55.3430	0.0138	0.0270	0.0617	0.0687
3	I	MM-7	950.0	3,623.0	61.8540	0.0222	0.0422	0.0623	0.0785
4	I	MM-8	1.155,0	3,623.0	75.2014	0.0327	0.0317	0.0246	0.0518
5	II	MM-3	860.0	3,264.0	57.9760	0.1400	0.1010	0.1830	0.2516
6	II	MM-5	830.0	3,264.0	55.9536	0.0206	0.0133	0.0407	0.0475
7	II	MM-6	965.0	3,264.0	65.0545	0.0045	0.0337	0.0555	0.0651
8	II	MM-6a	970.0	3,264.0	65.3916	0.0202	0.0568	0.0711	0.0932
9	II	MM-7	960.0	3,264.0	64.7174	0.0174	0.0477	0.0708	0.0871
10	II	MM-9	855.0	3,264.0	57.6390	0.0200	0.1070	0.0245	0.1116
11	II	MM-10	880.0	3,264.0	59.3234	0.0729	0.0936	0.0156	0.1197
12	II	MM-10a	885.0	3,264.0	59.6614	0.0134	0.1530	0.0426	0.1594

#### ◆ Model 1

On the basis of data given in Table 2, the soil oscillation law is calculated by the formula (4) - by the models 1 and 2. The calculation of the curve was carried out for values of reduced distances from  $R = 21.8117$  to  $R = 75.2014$ . Thus curve parameters were calculated enabling us determination the equation of soil oscillation in the form of :

$$v_l = 2.131,52 \cdot R^{-2,4410} \quad (21)$$

whereby the linear dependence between  $\log v$  and  $\log R$  was obtained, expressed by the equation (21), with the linear correlation coefficient  $r$  amounting:  $r = -0,8613$

Graphic survey of soil oscillation law is shown in Figure 2.



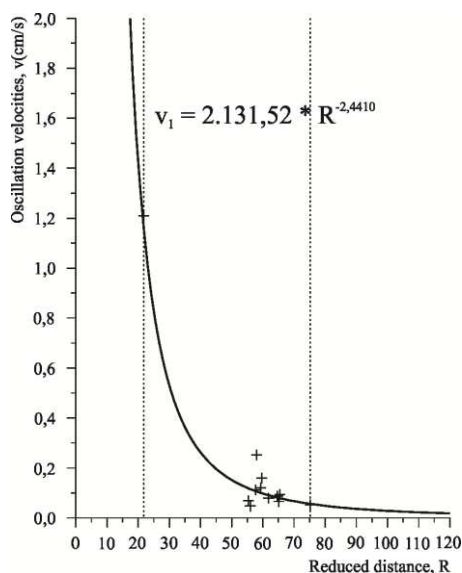


Figure 2 Graphic survey of soil oscillation law curve at the OP „Kovilovača”

◆ Model 2

$$v_2 = 38,9387 \cdot R^{-1,4227} \quad (22)$$

On the basis of the obtained equations of soil oscillation (21) and (22), it is possible to calculate values of soil oscillation velocities for corresponding reduced distances for models 1 and 2.

Table 3 presents the survey of reduced distances R, recorded oscillation velocities  $v_r$ , calculated oscillation velocities  $v_{i1}$ ,  $v_{i2}$  as well as the difference between recorded and calculated soil oscillation velocities for models 1 and 2.

Table 3 Survey of recorded and calculated soil oscillation velocities for models 1 and 2

No	R	$v_r$ [cm/s]	$v_{i1}$ [cm/s]	$v_{i2}$ [cm/s]	$v_r - v_{i1}$	$v_r - v_{i2}$
1*	21.8117	1.2100	1.1507	0.4851	0.0593	0.7249
2	55.3430	0.0687	0.1185	0.1290	-0.0498	-0.0603
3	61.8540	0.0785	0.0904	0.1101	-0.0119	-0.0316
4	75.2014	0.0518	0.0561	0.0834	-0.0043	-0.0316
5	57.9760	0.2516	0.1058	0.1207	0.1458	0.1309
6	55.9536	0.0475	0.1154	0.1270	-0.0679	-0.0795
7	65.0545	0.0651	0.0799	0.1025	-0.0148	-0.0374
8	65.3916	0.0932	0.0789	0.1017	0.0143	-0.0085
9	64.7174	0.0871	0.0809	0.1032	0.0062	-0.0161
10	57.6390	0.1116	0.1073	0.1217	0.0040	-0.0101
11	59.3234	0.1197	0.1001	0.1168	0.0196	0.0029
12	59.6614	0.1594	0.0987	0.1159	0.0607	0.0435

Based on the data in Table 3, a statistical analysis was carried out and the following values were obtained:

#### Model 1:

The curveline dependency index  $\rho_1$  between the reduced distance R and soil oscillation velocity is:

$\rho_1 = 0,9843$  (there is a *strong correlation* between R and v, given in the formula (22)).

The maximum difference between the recorded and calculated oscillation velocities of the soil  $\epsilon_{\max} = \max|\epsilon_i|$ , amounts:

$$\epsilon_{\max 1} = 0.1458; S_1 = 0.0552;$$

$$3S_1 = 0.1656.$$

As there is  $\epsilon_{\max 1} < 3S_1$ , supposed *functional relationship is accepted as a good one*.

#### Model 2:

$\rho_2 = 0.8513$  – for 11 values (there is a *strong correlation* between R and v, given in the formula (23)).

$$\epsilon_{\max 2} = 0.1309; S_2 = 0.0577;$$

$$3S_2 = 0.1713$$

$\epsilon_{\max 2} < 3S_2$  (supposed *functional relationship is accepted as a good one*).

## 5 CONCLUSION

To establish the relationship between the oscillation velocity of the rock mass and basic parameters affecting its magnitude, being: the amount of explosive, the distance from the blast site, characteristics of the rock mass and the type of blasting, it is the equation of M. A. Sadovski that is used most commonly.

In this study, parameters n and K in the law of Sadovsky were determined in two ways – models in the given work environment. The first model is the usual method of least squares, and the second model is derived by applying the Lagrange's theorem. Thereby corresponding functions were obtained presenting the oscillation velocities of the rock mass depending on a reduced distance. The calculated corresponding indexes

of the curveline correlation point out that there is a highly strong curveline relationship between the reduced distance and oscillation velocity of the rock mass expressed in the obtained functions.

Comparing values of the recorded oscillation velocities of the rock mass with the corresponding calculated ones, it can be seen that they are approximately the same. On the basis of obtained values of curveline dependency coefficients and the values of linear correlation coefficients between the reduced distance logarithm and the oscillation velocity logarithm, it can be concluded that both models can be used for calculating the oscillation velocity of the rock mass.

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## ODREĐIVANJE PARAMETARA ZAKONA OSCILOVANJA TLA UZ PRIMENU LAGRANŽOVE TEOREME NA PK „KOVILOVAČA“\*\*

### Izvod

Masovna miniranja, kao metoda eksploatacije mineralnih sirovina, imaju sve veću primenu, sa ciljem da se poveća količina odminirane mase i smanje troškovi proizvodnje. Povećanje količine odminirane mase zahteva upotrebu velikih količina eksploziva, što dovodi i do povećanja negativnih efekata miniranja. Pod negativnim efektima miniranja podrazumevamo seizmičko dejstvo miniranja, dejstvo vazdušnog talasa, zvučni efekat, razbacivanje odminirane stenske mase itd.

Za ocenu, kontrolu i planiranje seizmičkog dejstva miniranja, neophodno je utvrditi zakon oscilovanja tla u pravcu minsko polje – objekti koji se štite. Jedna od najčešće korišćenih je jednačina M.A. Sadovskog, koja definiše zakon promene brzine oscilovanja tla u zavisnosti od rastojanja, količine eksploziva, uslova izvođenja miniranja i geoloških karakteristika tla. Jednačina Sadovskog određuje se na osnovu probnih miniranja za konkretnu radnu sredinu.

U ovom radu posebna pažnja posvećena je seizmičkom dejstvu miniranja. U skladu sa tim izvršena je analiza metode za određivanje parametara zakona oscilovanja tla, koji je predložio ruski profesor M. A. Sadovski. Za određivanje parametara u jednačini Sadovskog, pored uobičajenog modela - metode najmanjih kvadrata, prikazan je još jedan model uz primenu Lagranžove teoreme. Pri tom je konstatovano da se oba modela mogu koristiti za izračunavanje brzine oscilovanja stenske mase.

**Ključne reči:** radna sredina, miniranje, seizmičko dejstvo, brzina oscilovanja, zakon oscilovanja tla

### 1. UVOD

Kao vezu između brzine oscilovanja tla i osnovnih parametara koji utiču na njenu veličinu, a to su: količina eksploziva, rastojanje od mesta miniranja, osobina stenskog materijala i način izvođenja miniranja, najčešće se koristi jednačina M. A. Sadovskog, gde je brzina oscilovanja  $v$  data u obliku:

$$v = K \cdot R^{-n},$$

gde  $R$  predstavlja redukovano tj. svedeno rastojanje, a parametri  $K$  i  $n$  uslovljeni su

karakteristikama tla i uslovima miniranja. Pri tome je  $v$  opadajuća i konveksna funkcija promenljive  $R$ .

Primenom zakona oscilovanja stenske mase pri miniranju omogućava se da se za svako miniranje unapred odredi brzina oscilovanja tla, a miniranja se u pogledu seizmičkog dejstva stavljaju pod kontrolu, što pruža mogućnost da se veličina potresa za svako sledeće miniranje unapred planira [2]. Na taj način smanjuju se negativni efekti miniranja. Pod negativnim efektima mini-

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\*\* Rad je proistekao iz projekta broj 33025 i 33029 koji je finansiran sredstvima Ministarstva za prosvetu i nauku Republike Srbije, a čiji je realizator Rudarsko-geološki fakultet, Univerziteta u Beogradu

ranja osim seizmičkog dejstva miniranja podrazumevamo i dejstvo vazdušnog talasa, zvučni efekat, razbacivanje odminirane stenske mase itd. Na taj način povećava se efikasnost proizvodnje i ujedno štite građevinski i rudarski objekti u okolini mesta miniranja, kao i životna sredina.

## 2. ZAKON OSCILOVANJA TLA

Za uspostavljanje korelacione veze između brzine oscilovanja i tri osnovna parametra koji utiču na njenu veličinu: količine eksploziva, osobine stenskog materijala i rastojanja, u svetu je razvijeno više modela. Jedan od najčešće korišćenih modela je jednačina Sadovskog, koja definiše zakon promene brzine oscilovanja tla u zavisnosti od rastojanja, količine eksploziva i načina izvođenja miniranja [10]. Tako definisan zakon pruža mogućnost da odredimo seizmičko dejstvo miniranja u pravcu nekog objekta ili naselja, pri čemu se koristi veza između brzine oscilovanja tla i posledica koje se mogu odraziti na objekte.

Jednačina M. A. Sadovskog data je u obliku:

$$v = K \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} \quad (1)$$

gde je:

v – brzina oscilovanja tla [cm/s],

K - koeficijent koji je uslovljen karakteristikama tla i uslovima miniranja, a određuje se terenskim merenjima,

$$v = K_v \cdot \left( \frac{\sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}}}{r} \right)^n = K_v \cdot \left( \frac{\sqrt[3]{3}}{\sqrt[3]{4 \cdot \pi}} \right)^n \cdot \left( \frac{\sqrt[3]{Q}}{r} \right)^n = K_v \cdot K_1 \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot \left( \frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot R^{-n}$$

gde je stavljeno:

$$\left( \frac{\sqrt[3]{3}}{\sqrt[3]{4 \cdot \pi}} \right)^n = K_1; K_v \cdot K_1 = K; \frac{r}{\sqrt[3]{Q}} = R$$

i gde je:

n - eksponent koji je uslovljen karakteristikama tla i uslovima miniranja, a određuje se takođe terenskim merenjima,

r - rastojanje od mesta miniranja do mesta opažanja [m],

Q - količina eksploziva [kg].

### 2.1. Izvođenje jednačine zakona oscilovanja stenske mase

Jednačina Sadovskog izvedena je iz uslova: ako se radijus punjenja  $r_0$  i rastojanje od mesta izvođenja miniranja do mesta opažanja  $r$  povećavaju u istoj ili približno istoj razmeri brzina oscilovanja tla  $v$  ostaje ista [1], tj. da je:

$$v = K_v \cdot \left( \frac{r_0}{r} \right)^n, \quad (2)$$

Radijus eksplozivnog punjenja  $r_0$  i količina eksploziva  $Q$  vezani su jednačinom:

$$Q = \frac{4}{3} \cdot \pi \cdot r_0^3,$$

odakle je:

$$r_0 = \sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}}. \quad (3)$$

Zamenom vrednosti  $r_0$  iz jednačine (3) u jednačini (2) dobijamo:

R - redukovano rastojanje ili svedeno rastojanje koje predstavlja rastojanje od mesta miniranja do mesta opažanja svedeno na količinu eksploziva,

$$\text{a dato je u obliku } R = \frac{r}{r_0}$$

Na taj način dobili smo zakon oscilovanja stenske mase tj. jednačinu Sadovskog u obliku:

$$v = K \cdot R^{-n} \quad (4)$$

## 2.2. Modeli određivanja parametara zakona oscilovanja tla

U jednačini (4) javljaju se dva parametra K i n, koje treba odrediti za konkretnu radnu sredinu i pri određenim uslovima miniranja. S obzirom na svojstvo zakona oscilovanja stenske mase, moguće je parametre K i n odrediti na više načina tj. modela, koristeći pri tome vrednosti dobijene eksperimentalnim merenjima.

### ♦ Određivanje parametara po modelu 1

Za dobijanje parametara (K) i (n) uglavnom se koristi metoda najmanjih kvadrata, koja predstavlja uobičajeni model [3].

### ♦ Određivanje parametara po modelu 2

U zakonu brzine oscilovanja stenske mase datog po jednačini (4) u obliku:

$$v = K \cdot R^{-n} \quad (4)$$

parametar n možemo odrediti uzastopnim aproksimacijama uz primenu Lagranžove teoreme [8].

Ako zakon brzine oscilovanja stenske mase dat jednačinom (4) diferenciramo po R dobićemo:

$$v' = -n \cdot K \cdot R^{-n-1},$$

tj.

$$v' = -\frac{n}{R} \cdot K \cdot R^{-n}. \quad (5)$$

Imajući u vidu (4), jednačina (5) se svodi na:

$$v' = -\frac{n \cdot v}{R}. \quad (6)$$

Iz (6) nalazimo:

$$n = -v' \cdot \frac{R}{v}. \quad (7)$$

Ako bismo našli izvod v' u nekoj tački R<sub>c</sub>, onda iz jednačine (7) možemo da odredimo n. Vrednost izvoda v' možemo odrediti pomoću neke od formula za numeričku diferencijaciju. U tu svrhu često se koristi formula Stirlinga za slučaj ekvidistantnih vrednosti za promenljivu R. Pri tome je potrebno da korak h bude mali broj što na primerima miniranja u rudarstvu nije slučaj.

Ranija ispitivanja su pokazala da se vrednost parametra n kreće uglavnom u intervalu od 1 do 3, najčešće u intervalu od 1 do 2.

Za određivanje parametra n koristimo Lagranžovu teoremu koja glasi:

Neka je funkcija f(x) neprekidna na [a,b] i diferencijabilna u intervalu (a, b). Tada postoji bar jedno c,

$$a < c < b,$$

za koje je:

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

U našem slučaju, s obzirom da je brzina oscilovanja neprekidna, opadajuća i konveksna funkcija na [R<sub>1</sub>, R<sub>s</sub>] i diferencijabilna u intervalu (R<sub>1</sub>, R<sub>s</sub>), za nju važi Lagranžova teorema:

$$\frac{v(R_s) - v(R_1)}{R_s - R_1} = v'(R_c),$$

$$R_1 < R_c < R_s. \quad (8)$$

Imajući u vidu formule (4) i (5), formula (8) se svodi na:

$$\frac{k \cdot R_s^{-n} - k \cdot R_1^{-n}}{R_s - R_1} = -k \cdot n \cdot R_c^{-n-1}, \quad (9)$$

tj. na:

$$\frac{R_s^{-n} - R_1^{-n}}{R_s - R_1} = -n \cdot R_c^{-n-1},$$

$$R_1 < R_c < R_s. \quad (10)$$

Iz jednačine (10) dobijamo:

$$R_c = \left( \frac{n \cdot R_s - R_1}{R_1^{-n} - R_s^{-n}} \right)^{\frac{1}{n+1}},$$

$$R_1 < R_c < R_s. \quad (11)$$

U praksi se može uzeti da je  $R_1$  najmanje redukovano rastojanje, a  $R_s$  najveće redukovano rastojanje koje je posmatrano prilikom miniranja tj. merenja.

U formuli (8) vrednosti  $v(R_s)$ ,  $v(R_1)$ ,  $R_s$  i  $R_1$  uzimamo iz tabele eksperimentalnih podataka, tako da imamo:

$$\frac{v_s - v_1}{R_s - R_1} = v'(R_c). \quad (12)$$

Na ovaj način nalazimo izvod  $v'(R_c)$  u tački  $R_c$ , gde je  $R_c$  dato pomoću formule (11).

U praksi formulu (8) koristimo tako što vrednosti  $v(R_s)$  i  $v(R_1)$  uzimamo iz tabele dobijenih eksperimentalnih podataka za odgovarajuća redukovana rastojanja  $R_s$  i  $R_1$ , pri čemu za  $R_s$  uzimamo najveće redukovano rastojanje, a za  $R_1$  najmanje redukovano rastojanje za svaku posmatranu tabelu.

Za nalaženje vrednosti  $v'(R_c)$ , u skladu sa formulom (8), korišćemo formulu:

$$v'(R_c) = \frac{v_s - v_1}{R_s - R_1} \quad (13)$$

gde je:

$v_s$  - registrovana brzina oscilovanja stenske mase za najveće redukovano rastojanje  $R_s$ ,

$v_1$  - registrovana brzina oscilovanja stenske mase za najmanje redukovano rastojanje  $R_1$ .

Na ovaj način dobijamo  $v'(R_c)$ , a zatim  $R_c$  određujemo po formuli (11). U formuli (11) za određivanje vrednosti  $R_c$  javlja se parametar  $n$ . Za početnu približnu vrednost parametra  $n$  uzecemo da je:

$$n = n_0 = 1,5.$$

Sada, prema formuli (11), za  $n = 1,5$  dobijamo:

$$R_c = \left( \frac{1,5 \cdot R_s - R_1}{R_1^{-1,5} - R_s^{-1,5}} \right)^{\frac{1}{2,5}}, \quad (14)$$

gde se dobijena vrednost  $R_c$  nalazi između vrednosti  $R_j$  i  $R_{j+1}$  datih u tabeli eksperimentalnih podataka, što znači:

$$R_j < R_c < R_{j+1}.$$

Sa ovako određenom vrednošću za  $R_c$  treba da odredimo vrednost  $v_c = v(R_c)$ , pri čemu je:

$$v_j < v_c < v_{j+1}.$$

Za nalaženje vrednosti  $v_c = v(R_c)$  korišćemo interpolacionu formulu:

$$v_c = v_j + \frac{v_{j+1} - v_j}{R_{j+1} - R_j} \cdot (R_c - R_j), \quad (15)$$

Sa ovako nađenom vrednošću  $v_c$ , u skladu sa formulom (7), dobijamo parametar  $n$  kao:

$$n = -v' \cdot \frac{R_c}{v_c}, \quad (16)$$

koji ćemo označiti sa  $n_1$ , pa prema formuli (8) imamo:

$$n_1 = -v' \cdot \frac{R_c}{v_c}. \quad (17)$$

Sada, sa ovako nađenom vrednošću  $n_1$  koristimo formulu (11) za određivanje nove vrednosti  $R_c$ , gde umesto  $n$  stavljamo  $n_1$ , pri čemu nastavljamo prethodni postupak za nalaženje nove približne vrednosti  $n_2$  parametra  $n$ .

U praksi, posle nekoliko iteracija dobija se zadovoljavajuća vrednost parametra  $n$ .

Vrednost parametra  $K$  dobijamo iz formule (4), pri čemu je:

$$K = v \cdot R^n \quad (18)$$

U jednačini (18) za  $n$  uzimamo vrednost koju smo dobili na prethodni način.

Pri tome se koriste podaci za parove ( $R_m, v_m$ ),  $m = 1, 2, \dots, N$ , iz tabele eksperimentalnih podataka, tako da uzimamo:

$$K_1 = v_1 \cdot R_1^n$$

$$K_2 = v_2 \cdot R_2^n$$

...

$$K_N = v_N \cdot R_N^n,$$

pri čemu za parametar  $K$  uzimamo njegovu aritmetičku sredinu tj:

$$K = \frac{K_1 + K_2 + \dots + K_N}{N}. \quad (19)$$

Na ovaj način odredili smo model rešavanja zakona oscilovanja stenske mase, uzimajući:

$$v = K \cdot R^{-n}$$

koji važi za datu sredinu gde je izvršeno miniranje.

### 3. DEFINISANJE STATISTIČKIH KRITERIJUMA

Za ocenu stepena povezanosti između registrovanih (izmerenih) i izračunatih podataka u ovom radu koristili smo *koeficijent linearne korelacije*  $r$  [5] između logaritama redukovanog rastojanja  $R$  i logaritama brzine oscilovanja  $v$ . Pored toga, uzimali smo u obzir i *indeks krivolinijske zavisnosti*  $\rho$  [4] između redukovanog rastojanja  $R$  i brzine oscilovanja  $v$ .

Ocena stepena povezanosti dve promenljive [8] prema vrednostima indeksa krivolinijske zavisnosti  $\rho$  data je u narednom pregledu:

$0,0 < \rho < 0,2$  nikakva ili veoma slaba veza,

$0,2 < \rho < 0,4$  slaba povezanost,

$0,4 < \rho < 0,7$  značajna povezanost,

$0,7 < \rho < 1,0$  jaka ili vrlo jaka povezanost.

Isto važi i za apsolutnu vrednost koeficijenta linearne korelacije  $r$ .

Kao mera pogodnosti dobijene funkcionalne veze za date eksperimentalne podatke koristili smo i kriterijum „3S“ [6]. Ovaj kriterijum koristi kvadrate razlika između dobijenih eksperimentalnih podataka i izračunatih podataka za brzine oscilovanja  $v$ . Ako su te razlike redom  $\varepsilon_1, \varepsilon_2 \dots \varepsilon_N$ , tada je:

$$S = \sqrt{\frac{\varepsilon_1^2 + \varepsilon_2^2 + \dots + \varepsilon_N^2}{N}} \quad (20)$$

Prema ovom kriterijumu, za ocenu pogodnosti dobijene funkcionalne veze važe sledeći odnosi:

- ako je  $|\varepsilon_{\max}| \geq 3S$ , odbacuje se dobijena funkcionalna veza kao nepovoljna,
- ako je  $|\varepsilon_{\max}| < 3S$ , prihvata se funkcionalna veza kao dobra.

## 4. PRIKAZ MASOVNIH MINIRANJA NA PK „KOVILOVAČA“ - DESPOTOVAC

### 4.1. Opšte karakteristike površinskog kopa „Kovilovača“

Ležište krečnjaka "Kovilovača" ima izuzetno prostu geološku građu. Krečnjaci ove zone su masivni ili slojeviti, pri čemu su slojevi debljine 0,20–0,80 m, pravac pružanja im je SI-JZ i pad od oko 42° prema jugozapadu. Ove stene, u inženjersko-geološkom smislu pripadaju grupi vezanih stena, koje su ispucale i karstifikovane [7]. Dosadašnjim istražnim i eksploacionom radovima u ležištu nisu konstatovane značajne rupturne deformacije, koje bi bitno uticale na proces istraživanja i eksploatacije. Jedino se posle miniranja javljaju blokovi stena od 0,50 m<sup>3</sup>, koji bi se mogli pripisati uticaju manjih raseda i karstifikovanih pukotina u ležištu.

Ispitivanjem fizičko - mehaničkih osobina radne sredine dobijene su sledeće vrednosti:



- ugao unutrašnjeg trenja
- kohezija
- zapreminska težina
- čvrstoća na pritisak
- čvrstoća na zatezanje
- brzina longitudinalnih elastičnih talasa
- brzina transverzalnih elastičnih talasa
- dinamički modul elastičnosti
- dinamički Poasonov koeficijent

$$\begin{aligned} \varphi &= 31^{\circ} 35' \\ C &= 138,33 \text{ [kN/m}^2\text{]} \\ \gamma &= 26,26 \text{ [kN/m}^3\text{]} \\ \sigma_p &= 808,08 \text{ [daN/cm}^2\text{]} \\ \sigma_z &= 75,90 \text{ [daN/cm}^2\text{]} \\ c_p &= 6.661,00 \text{ [m/s]} \\ c_s &= 2.852,67 \text{ [m/s]} \\ E_{din} &= 62,46 \text{ [GN/m}^2\text{]} \\ \mu_{din} &= 0,39 \text{ [GN/m}^2\text{]} \end{aligned}$$

#### 4.2. Način izvođenja miniranja

Merenja seizmičkih potresa na PK „Kovilovača“ obavljena su pri miniranjima koja se izvode radi eksploatacije ležišta [9]. Izvedena su dva miniranja.

Kao eksploziv korišćen je balkanit 60/1500, i detonex 65/1500, kao i ANFO 70/1500. Aktiviranje eksploziva u bušotinama vršeno je nonel detonatorima i to sa usporenjima u bušotini od 500 ms, dok je

usporenje između bušotina na površini bilo od 25 ms i 42 ms. Aktiviranje nonel cevčice izvršeno je električnim detonatorom.

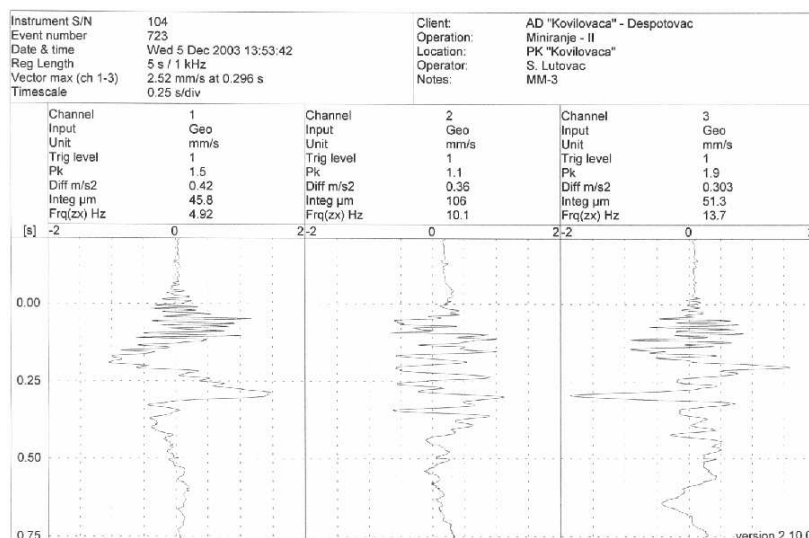
Osnovni podaci vezani za broj bušotina  $N_b$ , ukupnu količinu eksploziva  $Q_{uk}$ , maksimalnu količinu eksploziva po intervalu usporenja  $Q_i$ , ukupnu dubinu bušotina  $L_{uk}$ , i prosečnu dužinu čepa  $L_{pč}$ , dati su u tabeli 1.

**Tabela 1. Prikaz parametara miniranja**

Miniranje	$N_b$	$Q_{uk}$ [kg]	$Q_i$ [kg]	$L_{uk}$ [m]	$L_{pč}$ [m]
I	34	3623,0	108,0	842,0	2,2
II	7	3264,0	92,0	918,0	2,2

Na slici 1. prikazan je snimak brzine oscilovanja tla za miniranje broj II -

merno mesto broj 3.



**Sl. 1. Snimak brzine oscilovanja tla za miniranje II – MM3**

### 4.3. Proračun parametara zakona oscilovanja tla

Vrednosti rastojanja od mesta miniranja do mesta opažanja  $r$ , količina eksploziva  $Q$ , izračunate vrednosti redukovanih rastojanja  $R$ , registrovane vrednosti brzina

oscilovanja tla po komponentama  $v_t$ ,  $v_v$ ,  $v_l$  i rezultujuće brzine oscilovanja  $v_{rez}$  za miniranja I - II na ukupno deset mernih mesta MM date su u tabeli 2.

**Tabela 2.** Prikaz parametara miniranja i rezultata merenja

R.b.	Min. b.	MM	$r$ [m]	$Q$ [kg]	$R$	$v_t$ [cm/s]	$v_v$ [cm/s]	$v_l$ [cm/s]	$v_{rez}$ [cm/s]
1	I	MM-1	335,0	3.623,0	21,8117	0,1460	0,0517	1,2000	1,2100
2	I	MM-3	850,0	3.623,0	55,3430	0,0138	0,0270	0,0617	0,0687
3	I	MM-7	950,0	3.623,0	61,8540	0,0222	0,0422	0,0623	0,0785
4	I	MM-8	1.155,0	3.623,0	75,2014	0,0327	0,0317	0,0246	0,0518
5	II	MM-3	860,0	3.264,0	57,9760	0,1400	0,1010	0,1830	0,2516
6	II	MM-5	830,0	3.264,0	55,9536	0,0206	0,0133	0,0407	0,0475
7	II	MM-6	965,0	3.264,0	65,0545	0,0045	0,0337	0,0555	0,0651
8	II	MM-6a	970,0	3.264,0	65,3916	0,0202	0,0568	0,0711	0,0932
9	II	MM-7	960,0	3.264,0	64,7174	0,0174	0,0477	0,0708	0,0871
10	II	MM-9	855,0	3.264,0	57,6390	0,0200	0,1070	0,0245	0,1116
11	II	MM-10	880,0	3.264,0	59,3234	0,0729	0,0936	0,0156	0,1197
12	II	MM-10a	885,0	3.264,0	59,6614	0,0134	0,1530	0,0426	0,1594

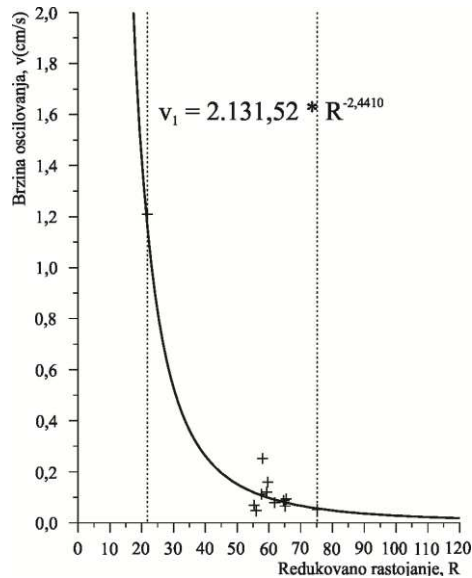
#### ♦ Model 1

$$v_l = 2.131,52 \cdot R^{-2,4410} \quad (21)$$

pri čemu je između  $\log v$  i  $\log R$  dobijena linearna zavisnost, izražena jednačinom (21) sa koeficijentom linearne zavisnosti  $r$  koji iznosi:  $r = -0,8613$

Grafički prikaz zakona oscilovanja tla dat je na slici 2.

Na osnovu podataka datih u tabeli 2 izračunava se zakon oscilovanja tla po formuli (4) - po modelima 1 i 2. Proračun krive izvršen je za vrednosti redukovanih rastojanja od  $R = 21,8117$  do  $R = 75,2014$ . Na taj način izračunati su parametri krive, koji omogućuju da se odredi jednačina oscilovanja tla u obliku:



Sl. 2. Grafički prikaz brzine oscilovanja tla na PK „Kovilovača“

◆ Model 2

$$v_2 = 38,9387 \cdot R^{-1,4227} \quad (22)$$

Na osnovu dobijenih jednačina oscilovanja tla (21) i (22), moguće je izračunati vrednosti brzina oscilovanja tla za odgovarajuća redukovana rastojanja za model 1 i 2.

U tabeli 3. dat je pregled redukovanih rastojanja R, registrovanih brzina oscilovanja tla  $v_r$ , izračunatih brzina oscilovanja tla  $v_{i1}$ ,  $v_{i2}$ , kao i razlika između registrovanih i izračunatih brzina oscilovanja tla za model 1 i 2.

Tabela 3. Prikaz registrovanih i izračunatih brzina oscilovanja tla za model 1 i 2

Redni broj	R	$v_r$ [cm/s]	$v_{i1}$ [cm/s]	$v_{i2}$ [cm/s]	$v_r - v_{i1}$	$v_r - v_{i2}$
1*	21,8117	1,2100	1,1507	0,4851	0,0593	0,7249
2	55,3430	0,0687	0,1185	0,1290	-0,0498	-0,0603
3	61,8540	0,0785	0,0904	0,1101	-0,0119	-0,0316
4	75,2014	0,0518	0,0561	0,0834	-0,0043	-0,0316
5	57,9760	0,2516	0,1058	0,1207	0,1458	0,1309
6	55,9536	0,0475	0,1154	0,1270	-0,0679	-0,0795
7	65,0545	0,0651	0,0799	0,1025	-0,0148	-0,0374
8	65,3916	0,0932	0,0789	0,1017	0,0143	-0,0085
9	64,7174	0,0871	0,0809	0,1032	0,0062	-0,0161
10	57,6390	0,1116	0,1073	0,1217	0,0040	-0,0101
11	59,3234	0,1197	0,1001	0,1168	0,0196	0,0029
12	59,6614	0,1594	0,0987	0,1159	0,0607	0,0435

Na osnovu podataka iz tabele 3. izvršena je statistička analiza i dobijene su sledeće vrednosti.

#### Model 1:

Indeks krivolinijske zavisnosti  $\rho_1$  između redukovanog rastojanja R i brzine oscilovanja tla v, iznosi:

$\rho_1 = 0,9843$  (postoji vrlo jaka povezanost između R i v, data u formuli (22)).

Maksimalna razlika između registrovanih i izračunatih brzina oscilovanja tla  $\epsilon_{\max}$   $\hat{=} \max|\epsilon_i|$ , iznosi:

$$\epsilon_{\max 1} = 0,1458; S_1 = 0,0552;$$

$$3S_1 = 0,1656.$$

Pošto je  $\epsilon_{\max 1} < 3S_1$ , pretpostavljena funkcionalna veza se prihvata kao dobra.

#### Model 2:

$\rho_2 = 0,8513$  – za 11 podataka (postoji vrlo jaka povezanost između R i v, data u formuli (22)).

$$\epsilon_{\max 2} = 0,1309; S_2 = 0,0577;$$

$$3S_2 = 0,1713.$$

$\epsilon_{\max 2} < 3S_2$  (pretpostavljena funkcionalna veza prihvata se kao dobra).

### 5. ZAKLJUČAK

Za uspostavljanje veze između brzine oscilovanja stenske mase i osnovnih parametara koji utiču na njenu veličinu, a to su: količina eksploziva, rastojenje od mesta miniranja, osobina stenskog materijala i način izvođenja miniranja, najčešće se koristi jednačina profesora M. A. Sadovskog.

U ovom radu parametri n i K u zakonu Sadovskog određivani su na dva načina – modela u datoj radnoj sredini. Prvi model predstavlja uobičajenu metodu najmanjih kvadrata, a drugi model je izveden uz primenu Lagranžove teoreme. Pri tome su dobijene odgovarajuće funkcije kojima su

predstavljene brzine oscilovanja stenske mase u zavisnosti od redukovanog rastojanja. Izračunati odgovarajući indeksi krivolinijske korelacije pokazuju da između redukovanog rastojanja i brzine oscilovanja stenske mase postoji vrlo jaka krivolinijska veza izražena dobijenim funkcijama.

Upoređujući vrednosti registrovanih brzina oscilovanja stenske mase sa odgovarajućim izračunatim vrednostima, vidimo da one imaju približno iste vrednosti. Na osnovu dobijenih vrednosti koeficijenata krivolinijske zavisnosti i vrednosti koeficijenata linearne korelacije između logaritama redukovanog rastojanja i logaritama brzine oscilovanja zaključujemo da se oba modela mogu koristiti za izračunavanje brzine oscilovanja stenske mase.

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